Thinking and Emotions Exhibited in Posing and Modeling Processes

Abstract

This study examines the impact of problem-posing on students' emotions during mathematical modeling tasks. Mathematical modeling involves posing and solving real-world problems using mathematical methods, enhancing students' understanding and engagement. However, the emotional responses elicited during these tasks are not well studied. Emotions significantly influence student engagement and learning efficacy, with AI-based facial recognition offering a novel method for their analysis. This research investigates how emotional experiences influence the outcomes of problem-posing and modeling processes. The study aims to bridge the gap in the literature by integrating problem posing into modeling tasks, providing a holistic view of the modeling process, and employing advanced emotion analysis to assess student emotions.

Keywords Problem Posing, Mathematical Modeling, Emotions, Facial Recognition

1 Introduction

Mathematical modeling, the activity of using mathematical knowledge and methods to interpret, represent, and solve practical problems, is of great importance in mathematics education (Blum, 2011; Kaiser, 2017). It has been recognized for its ability to deepen students' understanding of both the real world and mathematics (Abassian et al., 2020). Mathematical modeling involves two key components: posing a mathematical problem from the real world and solving it (Pollak, 2003). Although existing research underscores how problem posing involves identifying something important in the real world that we want to know, understand, and do (Geiger et al., 2022; Niss et al., 2007; Pollak, 2003), problem posing is not usually included in modeling tasks in modeling research (Cai et al., 2024).

In particular, the emotional responses elicited in problem posing and modeling have not been thoroughly investigated (e.g., Cai & Leikin, 2020). Emotions in educational settings are essential to indicate student engagement and learning efficacy (Pekrun & Linnenbrink-Garcia, 2012). Emotions such as anxiety or enjoyment can impact a student's ability to tackle complex problems, suggesting a nuanced interplay between emotion and cognition in mathematics education (Immordino-Yang & Damasio, 2007; Schukajlow et al., 2017). In previous research, emotion recognition methods primarily relied on questionnaires (Bieleke et al., 2023; Kanefke & Schukajlow, 2024). Now, with advances in artificial intelligence (AI), we can attempt to use sophisticated emotion analysis tools to analyze students' emotions more objectively and precisely.

This research explores problem posing in mathematical modeling tasks, focusing on students' model development for their posed problems and emotional responses during these processes. We aim to understand how emotions impact outcomes in problem-solving and mathematical modeling.

This study is significant for three key reasons: First, it fills a crucial gap in the existing literature by integrating problem posing with mathematical modeling, offering a more holistic view of the modeling process. Second, by examining students' emotional responses during problem posing and modeling, this research deepens the understanding of the often-overlooked effective dimensions of mathematics education. Third, employing advanced AI-based emotion analysis provides a novel and objective method for assessing student emotions, potentially setting new standards in educational research.

2 Methods

2.1 Tasks

This study employed four mathematical modeling tasks covering topics in algebra, geometry, and statistics. The tasks included: (a) the Doorbell task (Cai et al., 2023; Cai & Hwang, 2003), (b) the Oil Tank task (Kawakami et al., 2015; Lamb et al., 2017; Matsuzaki & Saeki, 2013), (c) the Light and Shadow task, and (d) the Vehicles task. Furthermore, to investigate the impact of problem posing, we adapted prompts to create three versions of the questionnaire:

(a) Problem Solving (called Solving version): This version follows traditional mathematical modeling tasks with specific questions for students to answer and engage in mathematical modeling.

(b) Problem Posing (called Posing version): In this version, students are asked to pose their own questions related to the task, answer these, and engage in mathematical modeling, without predefined questions.

(c) Problem Posing with a sample question (called Posing with sample version): Like the Problem Posing version but includes 1-2 predefined questions. Students answer these first, then pose their own questions and proceed with mathematical modeling.

As detailed in the appendix and confirmed through discussions with researchers, this questionnaire is considered a reliable tool for our experiments.

2.2 Participants

This study involved six students, three arts majors, and three science majors, detailed in Table 1. Pseudonyms were used to protect privacy.

Data for this study was collected in a quiet indoor setting with only participants and the researcher present; students could use calculators and ask questions during the test. The data collection was divided into three parts:

(a) Recording of the Mathematical Modeling Process: The recording was done using an iPad and Apple Pencil, with Notability as the software.

(b) Recording of Students' Facial Expressions: The recording was done using an iPhone 15 Pro's rear wide-angle camera at 1080p and 29.97 fps, positioned 60 cm in front of the students. The device has been widely used for measuring students' emotions (Clayton et al., 2015).

(c) 30-Minute Semi-Structured Interview: The interview focused on the student's emotional responses throughout the modeling test and the stages that made the most significant impression on them.

In both parts (a) and (b), the recording started when the students first viewed the test questions and continued until they raised their hands to indicate the end of the test.

2.3 Data Coding

Since Pollak (1997) introduced the steps of mathematical modeling, the process has been increasingly explored by researchers. The term "modeling cycle" is frequently used in the literature to describe this process. Common examples of modeling cycles include those proposed by Blum and Leiß (2007) and Stillman (2011). These modeling cycles resemble Pollak's eight-step approach, and after comparison, we distilled the steps to the following: (a) identifying, proposing, and refining a problem from the real world; (b) constructing a mathematical model from the mathematical problem; (c) obtaining mathematical results to explain the model; and (d) using the mathematical model to interpret real-world phenomena.

We coded our test using the four listed in Table 2, employing a 1-second interval as the coding unit. New steps were coded unless marked N/A. Writing pauses were considered part of the ongoing step if related to the same problem thought process. Pauses exceeding 10 seconds were marked as N/A.

2.4 Processing Facial Video for Emotion Extraction

The study of human emotions is well established, featuring diverse measurement methods including emotional scales (Bieleke et al., 2023; Lane et al., 1990), electroencephalograms (EEGs; Kim et al., 2013), and magnetic resonance imaging (MRI; Wu et al., 2021). This research adopts a recently popular approach: AI-based facial emotion recognition. The advantages of this method are: (a) it enables real-time emotion measurement and tracking, and (b) it effectively minimizes physical harm to participants and requires less data collection equipment than EEGs and MRIs. In this study, we utilized Hume AI, a facial measurement tool with 48 different emotional dimensions (Hume-AI, 2024), which has been rigorously tested and proven reliable (Brooks et al., 2024; Cowen et al., 2021).

2.5 Benchmarking Emotion Transitions and Modeling Steps

To study the emotional changes of students during the mathematical modeling process, we mapped the students' modeling steps to emotional codes on a timeline, enabling the exploration of emotional variations across different stages of the modeling process.

3 Results

The results of this study were analyzed from two perspectives: the student's cognitive and emotional responses throughout the entire experiment and their performance during the problem-posing phase.

3.1 Overview

3.1.1 Modeling Process

As can be seen in Figure 1, the time for art students was generally shorter than that for science students, particularly in Step C. Art students took significantly less time (average: 513s) compared to science students (average: 1921s). This suggests that science students favored mathematical methods for problem-solving, which often require more time for calculations and validation.

In contrast, for the Posing and Posing with sample tasks, art and science students spent almost the same amount of time on Step A (art students average: 1017s; science students average: 936s). This indicates that both groups spent similar time in the initial problem-posing stage, suggesting comparable cognitive processes in understanding and framing a problem.

3.1.2 Emotional Responses

During the Solving task, the art student maintained a calm demeanor. In the first half of the task, she experienced emotions such as joy, love, and amusement, suggesting that students often feel happiness and a sense of success after solving or posing a question. However, she later showed increased boredom and tiredness. Conversely, the science student showed less emotional fluctuation, predominantly displaying calmness, sadness, tiredness, and boredom. Additionally, she experienced disappointment in the early stages of the task and a moment of joy as the task concluded.

In the Posing with sample task, the art student initially felt joy and positivity. As the task progressed, tiredness and sadness became more dominant. When encountering difficulties, she first showed concentration followed by boredom, with occasional disappointment and distress. Towards the end of the task, emotions such as interest, joy, and love emerged. The science student began with joy and interest but shifted to disappointment and confusion, accompanied by sadness and late-stage tiredness. Notably, during Step C, she experienced significant distress.

In the Posing task without predefined questions, the art student mainly experienced boredom. She exhibited concentration during Step A and calmness during the initial thinking phase. As the task progressed to Step B, she displayed sadness and increasing tiredness over time.

The science student began with joy, followed by calmness, concentration, and tiredness. He gradually experienced boredom and sadness. It is noteworthy that during Step A, he occasionally felt joy, love, and interest.

3.2 Performance during problem posing

An interesting phenomenon was observed during the problem-posing step among the art students, specifically in the Posing with sample task. As shown in Figure 2, one student initially responded to the first question with, "I can't do it," indicating her uncertainty or lack of confidence. Despite this, she eventually succeeded in posing three new questions of varying difficulty on her own.

Initially, after stating "I can't do it," the student's primary emotions shifted from concentration to boredom. However, when she returned to the question approximately 1,400 seconds later and began to pose her own questions, her emotions had evolved to tiredness, sadness, a small amount of boredom, concentration, and distress. Interestingly, she experienced joy and love when finished. This emotional progression indicates that art students, despite initial doubts, can engage in problem-posing tasks. The initial response and subsequent emotional shifts reflect common reactions to challenging tasks—hesitation followed by deep engagement and eventual satisfaction.

4 Discussion

These results suggest distinct emotional patterns among art and science students during mathematical modeling tests.

4.1 Impact of Task Format

In the Solving version, art students initially enjoyed the tasks, possibly due to the novelty or challenge. But later felt boredom and fatigued from repetition or difficulty. Conversely, science students maintained a more consistent emotional state, likely due to their familiarity with mathematical problem solving, with brief joy at the end suggesting relief or satisfaction upon completion.

In the Posing version, art students' initial concentration and later boredom, sadness, and tiredness suggest that the open-ended task was initially engaging but became increasingly difficult and frustrating. Meanwhile, science students initially enjoyed applying their analytical skills creatively, but like art students, as the task progressed, the lack of structure led to boredom and sadness.

In the Posing with sample version, art students initially felt joy due to clear task guidelines but experienced negative emotions like tiredness and sadness as challenges increased. The emergence of positive emotions towards the end indicates accomplishment or relief. For science students, increasing disappointment and confusion might reflect the challenge of creating new problems, diverging from their typical structured problem-solving methods. Significant distress in Step C underscores the difficulty in generating mathematical models without predefined parameters.

4.2 Difference between Art and Science Students

The emotional patterns and problem-solving approaches of art and science students varied significantly. Art students engaged more initially but lost interest with increasing task difficulty. Conversely, science students showed more consistent emotions but struggled with unstructured tasks, indicating the impact of task familiarity and structure on performance and emotions. Art students tended to approach problems through intuitive and holistic processing, moving quickly through problem-solving steps but spending more time on problem-posing. Science students prefer systematic approaches, taking longer on computations but quicker in problem identification. Recognizing these disciplinary differences is key to tailoring educational methods in mathematical modeling, affecting both learning effectiveness and emotional experience.

4.3 Methodological Contribution

This study significantly advances methodology by using AI-based tools for real-time emotion analysis in mathematical education research. It maps emotional responses to different stages of mathematical modeling, revealing how emotions align with various cognitive engagements. This approach offers a replicable framework for future studies in other educational settings. Furthermore, employing AI tools for emotion analysis enhances the objectivity, precision, and depth of data on student engagement and learning experiences, potentially revolutionizing educational research.

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Table 1: Participant Characteristics Table

NO.	Pseudonym	Gender	Age	Education	Art or Science	Task Version
1	Emily	Female	25	Master student	Art	Solving
2	Kate	Female	26	PhD student	Art	Posing
3	Rebecca	Female	26	PhD student	Art	Posing with sample
4	Zoe	Female	30	PhD student	Science	Solving
5	Jeff	Male	25	PhD student	Science	Posing
6	Annette	Female	23	PhD student	Science	Posing with sample

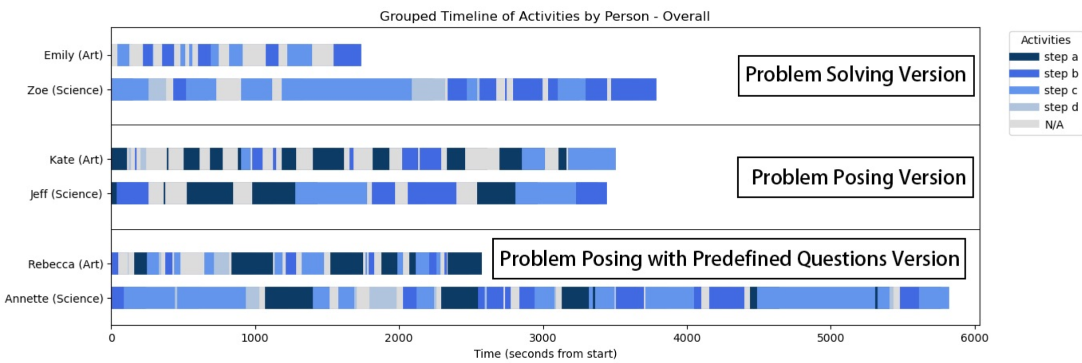


Figure 1: Overall Modeling Process.


2. 我国的一座石油精炼厂需要检查和维护一个巨大的重油储存罐，如右图。这个储油罐配备了一个螺旋形的楼梯，用于工作人员在检查时能够安全地沿罐体上下移动。然而，由于安全规定，石油精炼厂限制非安全检查人员进入罐区，维修团队无法直接测量楼梯的实际长度。通过与精炼厂的安全管理员沟通我们了解到罐体的直径和高度均为 10 米，楼梯与地面的夹角为 30°。

5m

10m

30°

（1）你可以利用这个尺寸数据，帮助维修团队需要计算螺旋楼梯的长度吗，以便他们可以准确计算所需材料并设计楼梯的加固方案。

不可以，我不会做

（2）根据上述场景，你还能提出哪些可以使用题目中的信息解答的数学问题？请提出一个简单的、一个中等难度的以及一个困难的数学问题。

困难：这个储油罐的容积是多少？

简单：储油罐占地多少平方米？

中等：如果要在罐体1米处修建隔离带，长度为多少米？

Figure 2: Partial Response from the Art Student - Posing with Sample Task.

Table 2: Mathematical Modeling Steps Coding Table

Step	Describe	How to use	e.g. in Table 3.
a	Identifying, proposing, and refining a problem from the real world	If the student writes down a problem such as "How many guests will arrive when the doorbell rings for the fifth time?" this is considered to be identifying and proposing a problem.	(a)
b	Constructing a mathematical model from the problem	When the student translates the problem into a mathematical model, for example, by using equations, diagrams, charts, or other mathematical tools to represent the observed real-world situation, this is constructing a mathematical model.	(b)
c	Obtaining mathematical results to explain the model	When the student performs calculations within the established mathematical model or problem, such as "$1 + 3 + 5 + \ldots + 19 = 100$", this is obtaining mathematical results to explain the model.	(c)
d	Using the mathematical model to interpret real-world phenomena	When the student explains the practical significance of the mathematical model, such as "$y = 2x - 1$, where x is the number of times the doorbell rings, and f is the number of guests," this is using the mathematical model to interpret real-world phenomena.	(d)
N/A	Cannot be identified from the video, typically characterized by the page remaining static with no writing occurring.	If a segment of the recording cannot be classified into any of the above steps and the page remains static with no writing for more than 10 seconds, it is coded as N/A. This usually indicates that the student is thinking, but we cannot determine the specific content of their thoughts.	N/A

Table 3: Example of the data coding

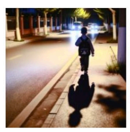
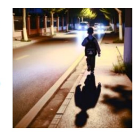
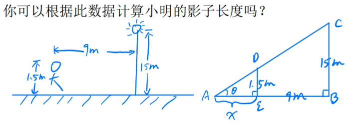
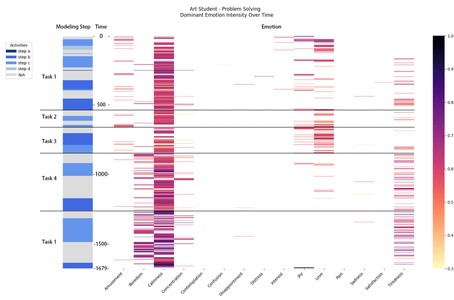
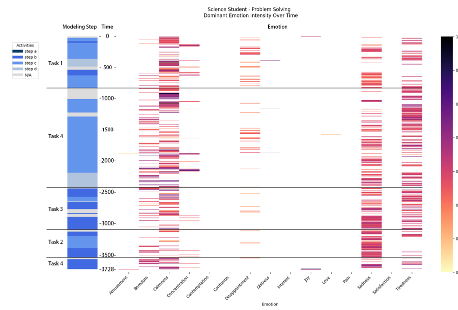
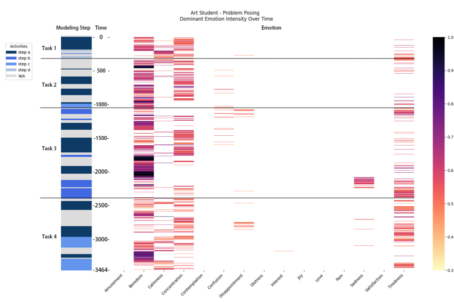
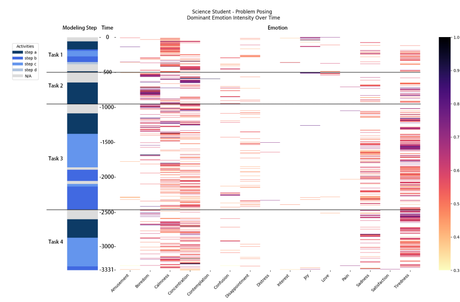
	Before	After
(a)	1. 李明正在正在组织一个聚会。每当门铃响起，都会有客人进入。 第一次门铃响时，进入了 1 位客人； 第二次门铃响时，进入了 3 位客人； 第三次门铃响时，进入了 5 位客人； 第四次门铃响时，进入了 7 位客人； 依此类推... （1）根据上述场景，你能否根据题目提供的信息设计出数学问题？（请设计一个简单的、一个中等难度的以及一个困难的数学问题。）	1. 李明正在正在组织一个聚会。每当门铃响起，都会有客人进入。 第一次门铃响时，进入了 1 位客人； 第二次门铃响时，进入了 3 位客人； 第三次门铃响时，进入了 5 位客人； 第四次门铃响时，进入了 7 位客人； 依此类推... （1）根据上述场景，你能否根据题目提供的信息设计出数学问题？（请设计一个简单的、一个中等难度的以及一个困难的数学问题。） 简单：第五次门铃响时，会进入几位客人？ 中等：第五次门铃响后，一共来了几位客人？ 困难：第一百次门铃响后，一共来了几位客人？
(b)	3. 小明放学回家时，行走在路旁，路灯将街道照的明亮，小明的影子也清晰可见，这让他觉得在回家的路上有影子的陪伴也不那么孤单了。  （1）假设路灯高 15 米，小明身高 1.5 米，小明距离路灯的距离为 9 米，你可以根据此数据计算小明的影子长度吗？ （2）根据上述场景，你还能提出哪些可以使用题目中的信息解答的数学问题？请提出一个简单的、一个中等难度的以及一个困难的数学问题。	3. 小明放学回家时，行走在路旁，路灯将街道照的明亮，小明的影子也清晰可见，这让他觉得在回家的路上有影子的陪伴也不那么孤单了。  （1）假设路灯高 15 米，小明身高 1.5 米，小明距离路灯的距离为 9 米，你可以根据此数据计算小明的影子长度吗？  （2）根据上述场景，你还能提出哪些可以使用题目中的信息解答的数学问题？请提出一个简单的、一个中等难度的以及一个困难的数学问题。
(c)	（2）某次门铃响后，此时屋内恰好有 100 位客人，请问门铃共响了几次？请解释你是如何找到答案的；	（2）某次门铃响后，此时屋内恰好有 100 位客人，请问门铃共响了几次？请解释你是如何找到答案的； 忘记了求和公式！ $20\times2-1=39$ $1+3+5+7+9+\cdots+19=100$ $2\times10-1=19$
(d)	（3）你可以尝试构建一个数学模型，来解决与第（2）问中你选择的数学问题相似的同一类问题吗？ （注 1：数学模型可以是一个数学方程、表达式或函数图像等；注 2：解决同一类问题是指，改变某些条件，或引入某些新的条件，依旧可以使用该模型解决此类问题）	（3）你可以尝试构建一个数学模型，来解决与第（2）问中你选择的数学问题相似的同一类问题吗？ （注 1：数学模型可以是一个数学方程、表达式或函数图像等；注 2：解决同一类问题是指，改变某些条件，或引入某些新的条件，依旧可以使用该模型解决此类问题） $y=2x-1$ x为第几次门铃响，y为客人数量

Table 4: Time-Aligned Dominant Emotion Intensity and Task Coding Across Participants

	Art Student	Science Student
Problem Solving		
Problem Posing		
Problem Posing with predefined questions